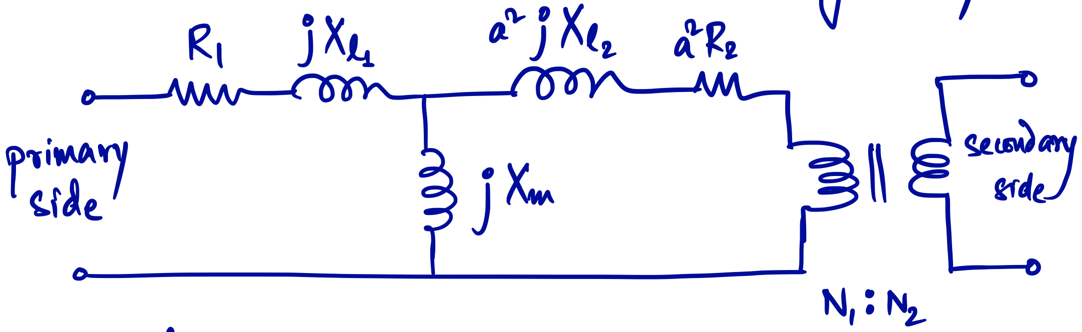
We will typically study transformers with sinusoidally varying currents & voltages.

Thus, our circuit model is now given by



Nomenclature :

X_{ℓ_1} = leakage reactance of primary coil/winding

$a^2 X_{\ell_2}$ = leakage " " secondary coil/winding, referred to the primary side.

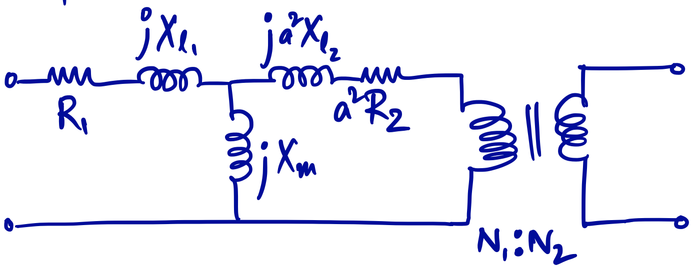
X_m = Magnetizing reactance.

R_1 = Resistance of primary coil

$a^2 R_2$ = Resistance of secondary coil, referred to the primary side.

Modeling a practical power transformer:

So far, we have derived an equivalent circuit representation of a transformer (with ideal transformer & linear elements) from first principles using magnetic circuits. Now, we turn towards modeling transformers in practice, i.e., account for non-idealities in the model.



This is what we have so far. Let's add some non-idealities to it.

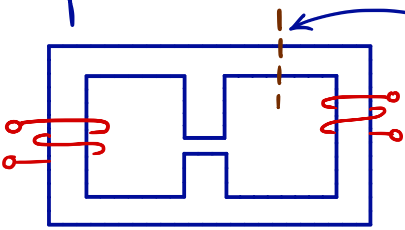
- What does R_1 & R_2 model?

They model "copper losses", the losses in the wires or coils wound around the core.

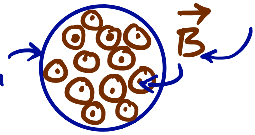
• Next, we model non-idealities in the core.

① Eddy current losses

Suppose the core is made of a solid piece of metal as shown.

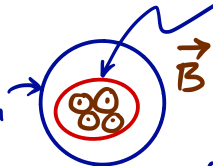


Let's look at a cross section of the core, where we overlay the magnetic flux.



changing magnetic field.

Now, consider a conceptual loop inside the core as shown.



Since the magnetic flux linked with this conceptual loop is changing, it will induce an "emf" around the loop.

An emf can drive a tiny current through the loop, if the core is made of a good electrical conductor. Recall that cores of transformers are made up of good magnetic materials, most of which are metals, that are also good electrical conductors.

$\Rightarrow$ $\underbrace{\text{Tiny currents}}$ flow through the core

called "eddy currents".

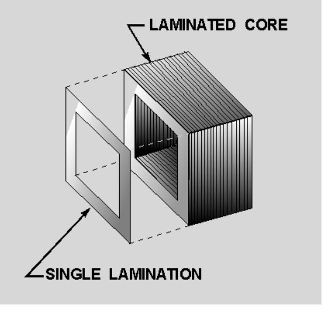
$\Rightarrow$ (Current^2 · resistance) losses dissipate as heat.

Eddy currents, though tiny, can add up through the core. Losses due to eddy currents can be shown to be $\underbrace{(K_{eddy})}_{\text{a constant}} \cdot \underbrace{f^2}_{\text{frequency}} \cdot \underbrace{B^2_{max}}_{\text{max. flux density.}}$

We will not derive this relation from first principles.

Q. Can you think of a mechanism to reduce eddy current losses?

A. Create the core by gluing together a collection of metallic sheets or "laminates". The idea is to reduce the possibilities of conducting loops inside the core through which eddy currents can flow.



Source: A post about eddy currents on Quora.

② Hysteresis losses:

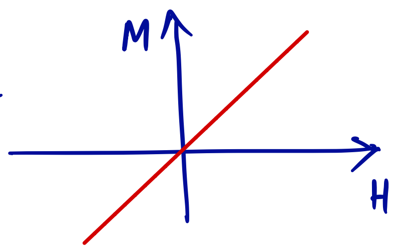
We have assumed so far that the core is magnetically linear, where

$$\underbrace{\vec{B}}_{\text{magnetic flux density}} = \underbrace{\mu}_{\text{magnetic permeability.}} \underbrace{\vec{H}}_{\text{magnetic field intensity}}$$

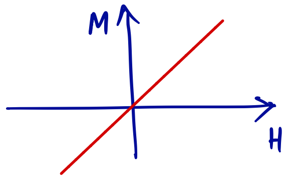
A more detailed model reads as follows:

$$\vec{B} = \underbrace{\mu_0}_{\text{magnetic permeability of air/vacuum}} \left(\vec{H} + \underbrace{\vec{M}}_{\text{magnetization.}} \right)$$

Linearity of magnetic material can be equivalently stated as $\vec{M} \propto \vec{H}$, i.e., the graph between M and H should look like

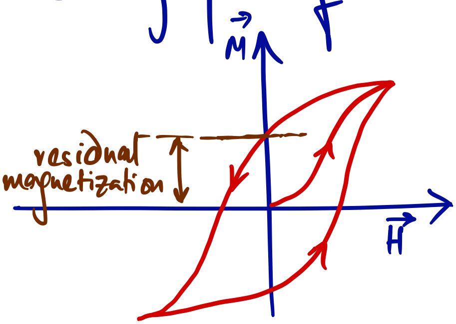


Notice what the linear graph means. As you



increase the current (creating a magnetic field), the material gets magnetized, and the magnetization disappears as you decrease the current back to zero. In other words, there is no residual magnetization after the current through the coil around the core is turned off. In practice, you always have some residual magnetization.

The graph of $\vec{H}$ v/s $\vec{M}$ often looks like as shown. It also depends on the "path", i.e., it depends on the magnetic history of the material.



This dependence of magnetization on the magnetic history of the material is called hysteresis. Hysteresis causes losses that are not modeled in assuming a linear magnetic material. Why does hysteresis lead to a loss? One way to think about it is that you need to drive additional current in the opposite direction to remove residual magnetization, and you spend this extra "effort" in every cycle when you run sinusoidal currents through the coils.

$$\text{Hysteresis losses} \approx (K_{hysteresis}) \cdot f \cdot B_{max}^{n}.$$

(we will not derive this expression.)

In this course, assume $n = 2$.

Eddy current losses + hysteresis losses (Call this core losses).

$$= (K_{eddy}) . f^2 B_{max}^2 + (K_{hysteresis}) . f . B_{max}^2$$

$$= K . B_{max}^2 ,$$

when operated at a constant frequency of $f = 60$ Hz.

$\therefore$ Core losses (Eddy current & hysteresis losses)

$$\propto B_{max}^2$$

$$\propto V_{max}^2 .$$

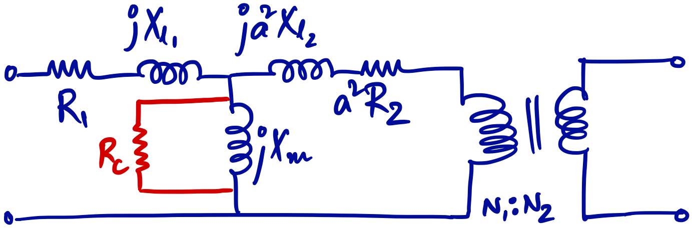
Why? $B_{max} \propto \phi_{max} \propto V_{max}$.

... from previous lecture.

We can model it by a resistor in a circuit model, i.e., core losses $= \dfrac{V_{max}^2}{R_c}$.

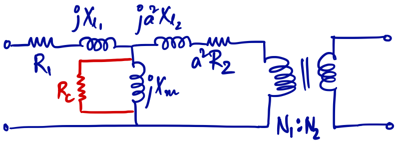
Let's add R_c to our ckt model.

- Circuit equivalent with core losses:

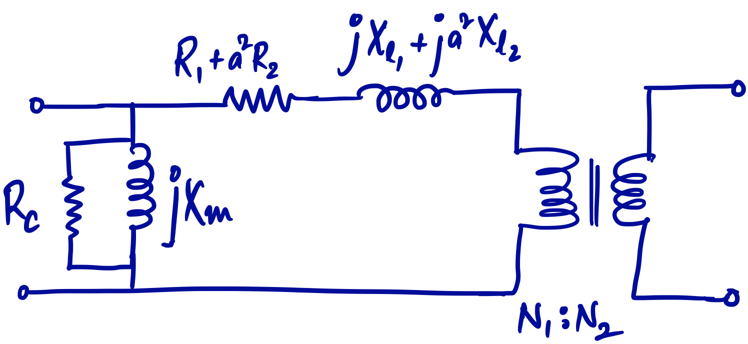


We do not compute R_c from first principles, but rather measure it from experiments.

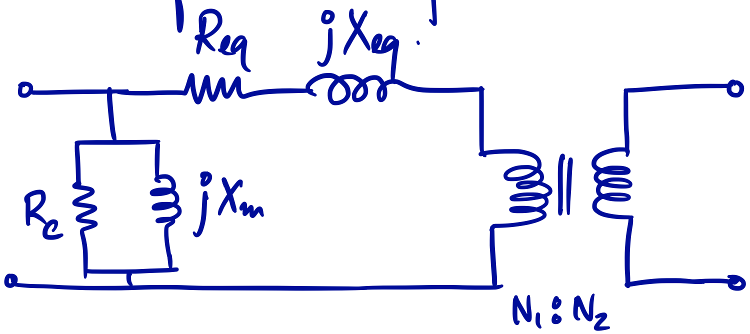
- Approximate circuit equivalent of a power transformer



approximately equivalent.



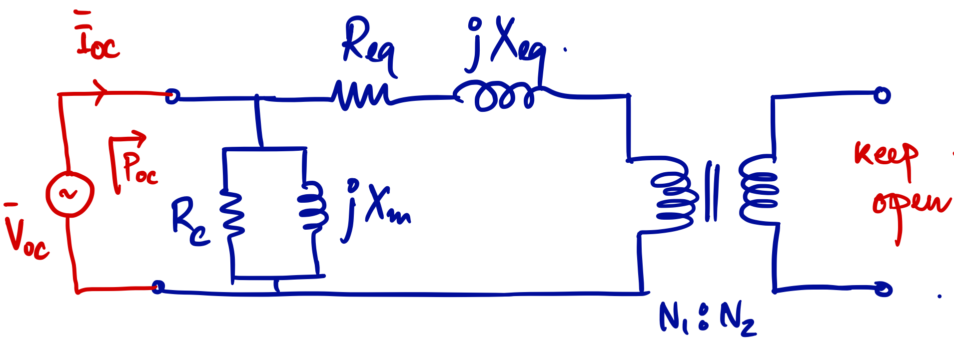
When are these two circuits approximately equivalent? When R_c & jX_m are "large" so that the current that flows through them is "small" compared to the currents flowing through the other parts of the circuit. Thus, we have arrived at an "equivalent circuit" model of a transformer:



Next on the agenda:

Design experiments to determine R_c, X_m, R_{eq}, X_{eq} ... open & short circuit tests.

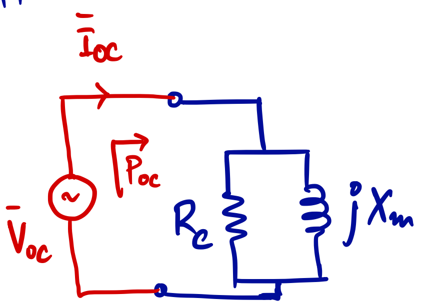
Open circuit test: (Finding R_c & X_m).

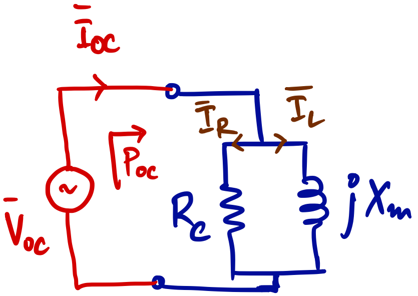


Connect a sinusoidal voltage source as shown, leaving the other end "open". Suppose

- we know $|\bar{V}_{oc}|$,
- we measure $|\bar{I}_{oc}|$, and the real power P_{oc} drawn by the transformer.
- Goal: Compute R_c, X_m.

Effective circuit is shown below. Since no current flows through the secondary side of the ideal transformer, no current can flow through the branch with R_{eq}, X_{eq}.





- P_{oc} (Real power) is only consumed by the resistive element.

$$\Rightarrow \quad \frac{|\bar{V}_{oc}|^2}{R_c} = P_{oc}$$

$$\Rightarrow \quad R_c = \frac{|\bar{V}_{oc}|^2}{P_{oc}} \quad \ldots (1)$$

- Notice that $\bar{I}_R = \dfrac{\bar{V}_{oc}}{R_c}$ and $\bar{I}_L = \dfrac{\bar{V}_{oc}}{jX_m}$.

$$\Rightarrow \bar{I}_R \perp \bar{I}_L. \quad \text{(Why?)}$$

- Also, $\bar{I}_{oc} = \bar{I}_R + \bar{I}_L$

$$\Rightarrow |\bar{I}_{oc}|^2 = |\bar{I}_R|^2 + |\bar{I}_L|^2.$$

$$\Rightarrow |\bar{I}_{oc}|^2 = \frac{|\bar{V}_{oc}|^2}{R_c^2} + \frac{|\bar{V}_{oc}|^2}{X_m^2}$$

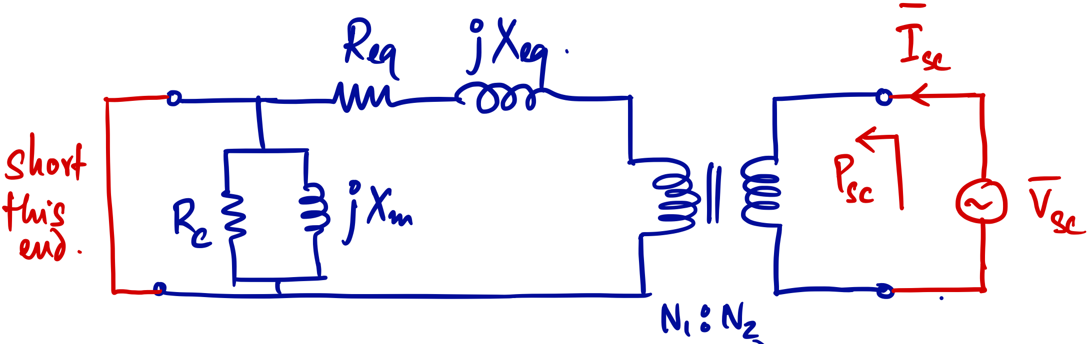
In this relation, you know $|\bar{V}_{oc}|$, $|\bar{I}_{oc}|$ and R_c.

$\vdots$

Calculate X_m!

$$\Rightarrow X_m = \left(\frac{|\bar{I}_{oc}|^2}{|\bar{V}_{oc}|^2} - \frac{1}{R_c^2}\right)^{-1/2}.$$

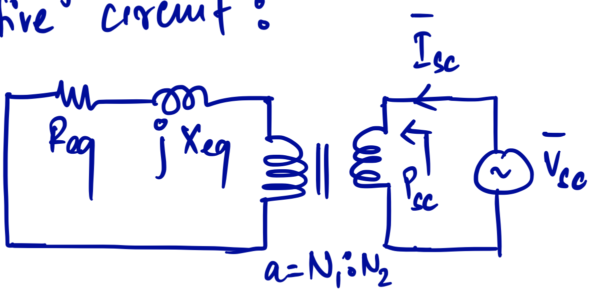
Short-circuit test: (Finding R_{eq}, X_{eq}).

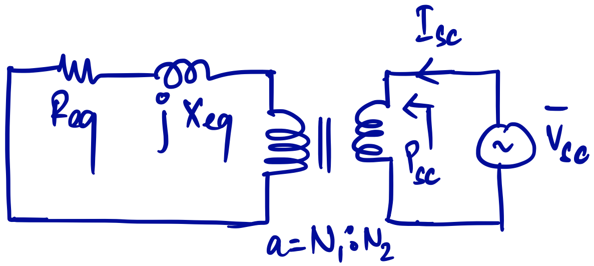


In this test, you short one end and put a voltage source at the other end.

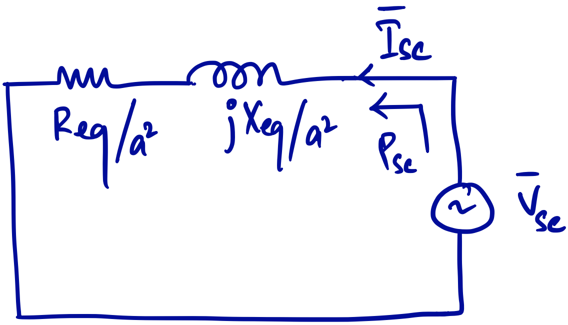
- we know $|\bar{I}_{sc}|$.
- we will measure $|\bar{V}_{sc}|$ and P_{sc}, where P_{sc} is the real power drawn by the transformer as shown.

Effective circuit:





Further simplify it by referring R_{eq}, X_{eq} to the source side.



- $P_{sc} = |\bar{I}_{sc}|^2 \dfrac{R_{eq}}{a^2}$

(why?)

$\Rightarrow R_{eq} = a^2 \cdot \dfrac{P_{sc}}{|\bar{I}_{sc}|^2}$.

- $\dfrac{1}{a^2}\sqrt{R_{eq}^2 + X_{eq}^2} = \left|\text{impedance of the circuit}\right| = \dfrac{|\bar{V}_{sc}|}{|\bar{I}_{sc}|}$

$$\Rightarrow X_{eq} = \sqrt{\left(\frac{a^2 |\bar{V}_{sc}|}{|\bar{I}_{sc}|}\right)^2 - R_{eq}^2}.$$

You don't need to remember the formulae.

Remember the key steps:-

- Real power is only consumed by resistors.
- Open ckt test: $\bar{I}_R \perp \bar{I}_L$.
- Short ckt test: $\dfrac{|\bar{V}_{sc}|}{|\bar{I}_{sc}|} = \left|\text{impedance referred to the source side}\right|$.

Some more conventions:

Open ckt test: $|\bar{V}_{oc}| =$ Rated voltage on the low voltage side of the transformer.

Short ckt test: $|\bar{I}_{sc}| =$ Rated current on the high voltage side of the transformer.

We choose these so that we require voltage sources with lower voltages.

What is "rated" voltage & current of a transformer? ... Next page!

Ratings of a transformer:

- $\underbrace{220\,V}_{\text{Rated voltage on the "low" voltage side}} / \underbrace{4400\,V}_{\text{Rated voltage on the "high" voltage side}}$, $\underbrace{2.5\,kVA \text{ transformer}}_{\text{Rated power}}$.

$$\text{Rated current on any side} = \frac{\text{Rated Power}}{\text{Rated voltage on the same side}}.$$

- Sometimes a "% Z" is also given.

For example, 10% Z means $|\bar{V}_{sc}| = 10\%$ of rated voltage on high voltage side.

In our example, $|\bar{V}_{sc}| = 10\%$ of $4400\,V$

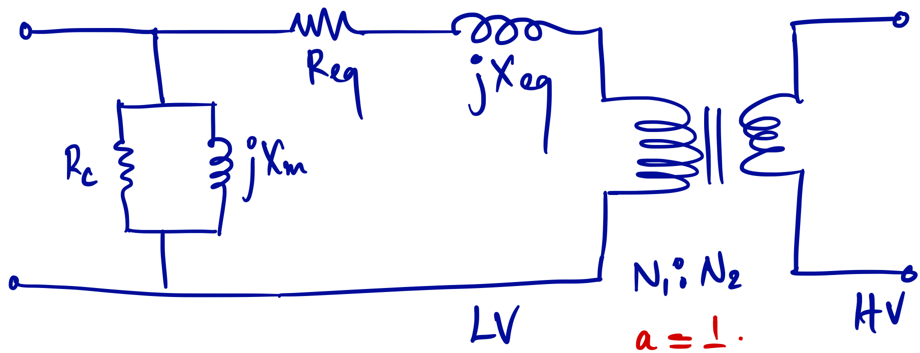
$= 440\,V$.

Voltage for short circuit test.

- Let's do an example: The results of OC & SC tests on a 440V/220V, 7.5 kVA transformer are given below:
  - $I_{oc} = 1\,A$, $P_{oc} = 50\,W$
  - $V_{sc} = 15\,V$, $P_{sc} = 60\,W$.

Compute R_c, X_m, R_{eq}, X_{eq} referred to the low voltage side.

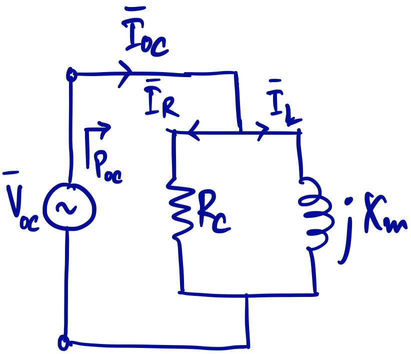
Solution:



$$a = \underbrace{\frac{1}{2}}_{\text{from the rating}}.$$

- Compute R_c, X_m from OC test:

$$|\bar{V}_{oc}| = \text{Rated voltage on low voltage side} = 220\,V.$$



- $P_{oc} = |\bar{V}_{oc}|^2 / R_c$

$$\Rightarrow R_c = |\bar{V}_{oc}|^2 / P_{oc} = (220V)^2 / 50W = 968\,\Omega.$$

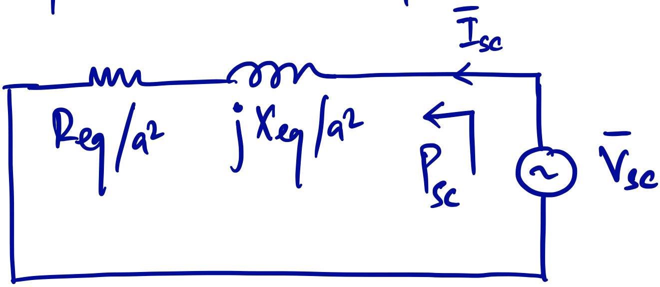
- $|\bar{I}_R| = |\bar{V}_{oc}| / R_c = 220V / 968\,\Omega = 0.23\,A.$

$$|\bar{I}_L|^2 + |\bar{I}_R|^2 = |\bar{I}_{oc}|^2. \quad \text{(Why?)}$$

$$\Rightarrow |\bar{I}_L| = \sqrt{(1A)^2 - (0.23A)^2} = 0.97\,A$$

$$\Rightarrow X_m = |\bar{V}_{oc}| / |\bar{I}_L| = 220V / 0.97\,A = 226\,\Omega.$$

• Compute R_{eq}, X_{eq} from SC test.



$$|\bar{I}_{sc}| = \text{Rated current on HV side} = \frac{\text{Rated power}}{\text{Rated voltage on HV side}}$$

$$= \frac{7.5\ \text{kVA}}{440\ \text{V}}$$

$$= 17\ \text{A}.$$

• $P_{sc} = |\bar{I}_{sc}|^2 \cdot R_{eq}/a^2.$

$$\Rightarrow R_{eq} = a^2 \cdot P_{sc}/|\bar{I}_{sc}|^2 = \left(\frac{1}{2}\right)^2 \cdot 60\text{W} \cdot \frac{1}{(17\text{A})^2} = 0.05\ \Omega.$$

• $\dfrac{|\bar{V}_{sc}|}{|\bar{I}_{sc}|} = \dfrac{1}{a^2} \cdot \sqrt{R_{eq}^2 + X_{eq}^2}.$

$$\Rightarrow X_{eq} = \left[\left(a^2 \frac{|\bar{V}_{sc}|}{|\bar{I}_{sc}|}\right)^2 - R_{eq}^2\right]^{1/2} = \left[\left(\frac{1}{4} \cdot \frac{15\text{V}}{17\text{A}}\right)^2 - (0.05\Omega)^2\right]^{1/2}$$

$$= 0.21\ \Omega$$

- If you were asked to report the answers referred to the HV side, multiply all of them with a^2, and report them.

Another example:

150 kVA, 2400 V / 240 V transformer has $R_{eq} = 0.425\ \Omega$, $X_{eq} = 0.914\ \Omega$, $R_c = 9931\ \Omega$, referred to the high voltage side. It suplies a load at rated voltage and current at 0.8 pf lagging on the low voltage side.

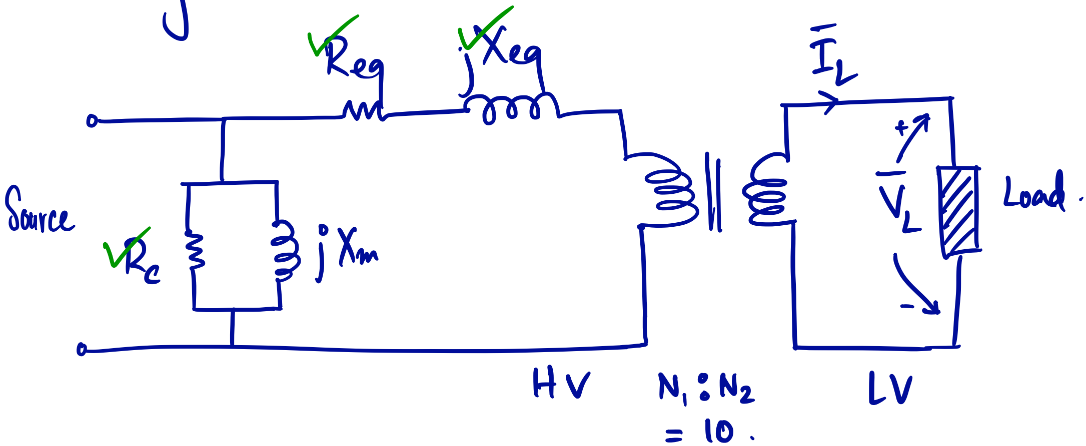
Find:

① Efficiency η of the transformer

② Voltage regulation provided by the transformer.

Both these terms are new. We'll learn through the example.

- Draw the approximate equiv circuit from the side, where data for the impedances are given.



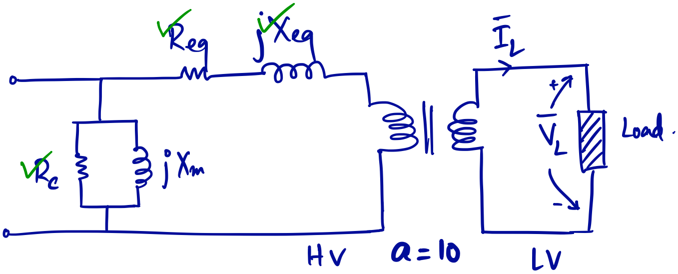
$$\text{Efficiency } (\eta) = \frac{\text{Power supplied to load}}{\text{Power drawn from Source.}}$$

$$= \frac{P_{load}}{P_{load} + P_{loss}}$$

P_{loss}: Losses in R_{eq} {Copper loss

\+ Losses in R_c {Core loss.

Let's calculate it for the example.



• Computing P_{Load} :

We know $|\bar{V}_L|$ = Rated voltage on the LV side = 240 V.

We know $|\bar{I}_L|$ = Rated current on the LV side $= \dfrac{\text{Rated power}}{\text{Rated voltage on LV side.}}$

$= \dfrac{150 \text{ kVA}}{240 \text{ V}}$

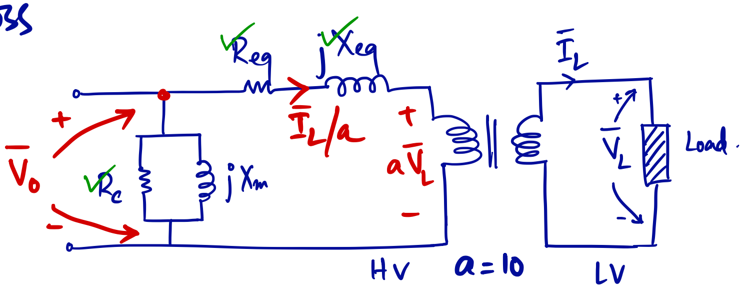
$= 625 \text{ A}.$

$\therefore P_{load} = |\bar{V}_L| \cdot |\bar{I}_L| \cdot \cos\theta$

$= 240\text{V} \cdot 625\text{A} \cdot 0.8$

$= 120 \text{ kW}.$

- Computing P_{loss}



$$P_{loss} = \underbrace{\left|\frac{\bar{I}_L}{a}\right|^2 \cdot R_{eq}}_{T_1} + \underbrace{\frac{|\bar{V}_0|^2}{R_c}}_{T_2}$$

- Computing T_1: $T_1 = \left(\frac{625A}{10}\right)^2 \cdot 0{\cdot}425\,\Omega$
$$= 1{\cdot}67 \text{ kW}.$$

- Computing T_2: We need $|\bar{V}_0|$.

Notice that $\bar{V}_0 - \left(\frac{\bar{I}_L}{a}\right)(R_{eq} + jX_{eq}) = a\cdot\bar{V}_L$

$$\Rightarrow \bar{V}_0 = \frac{\bar{I}_L}{a}(R_{eq} + jX_{eq}) + a\bar{V}_L.$$

$\bar{I}_L$ and $\bar{V}_L$ are phasors. You have to compute the complex number $\bar{V}_0$ and then calculate $|\bar{V}_0|$.

Without loss of generality, let $\angle \bar{V}_L = 0$.

Then, $\bar{V}_L = 240 \angle 0$ V.

You know $|\bar{I}_L|$. Compute $\angle \bar{I}_L$ from the power factor of the load.

$$\angle \bar{V}_L - \angle \bar{I}_L = \underbrace{+}_{\sim} \cos^{-1}(0.8) = 37°$$

because lagging power factor means current lags behind voltage in their phases.

$$\Rightarrow \angle \bar{I}_L = -37°.$$

$$\Rightarrow \bar{I}_L = 625 \angle -37° \text{ A}.$$

$$\therefore \bar{V}_0 = \frac{\bar{I}_L}{a}(R_{eq} + jX_{eq}) + a\bar{V}_L.$$

$$= \left(\frac{625 \angle -37° \text{ A}}{10}\right)(0.425\,\Omega + j\,0.914\,\Omega) + 10 \cdot 240 \angle 0° \text{ V}.$$

$$= 2.46 \angle 0.69° \text{ kV}.$$

$$\therefore T_2 = |\bar{V}_0|^2 / R_c = \frac{(2.46 \times 10^3 \text{ V})^2}{9931\,\Omega} = 0.61 \text{ kW}.$$

Efficiency $\eta = \dfrac{P_{load}}{P_{load} + \underbrace{P_{loss}}_{= T_1 + T_2}} \times 100\%$

$$= \frac{120 \text{ kW}}{120\text{kW} + 1.67\text{kW} + 0.61\text{kW}} \times 100\%$$

$= 98.14\%$ } No practical transformer has 100% efficiency.

- Voltage regulation is defined as the % change in voltage magnitude due to presence of load,

i.e.,

$$\text{voltage regulation} = \frac{\left|\text{Voltage with no load}\right| - \left|\text{voltage with load}\right|}{\left|\text{voltage with load}\right|} \times 100\%$$

Voltage with load $= |\bar{V}_L| = 240$ V.

Voltage without load $= |\bar{V}_0| = 246$ V (why?)

$\Rightarrow$ % voltage regulation $= \dfrac{246 - 240}{240} \times 100\% = 2.5\%$.